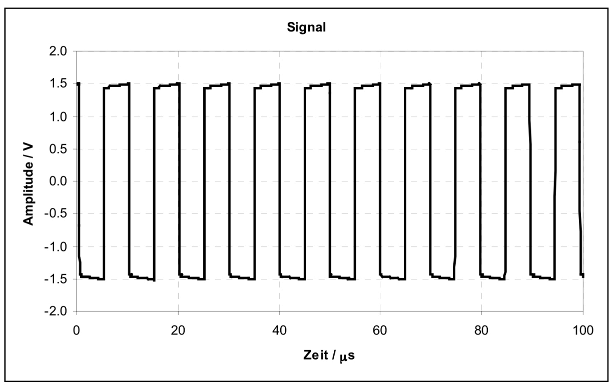
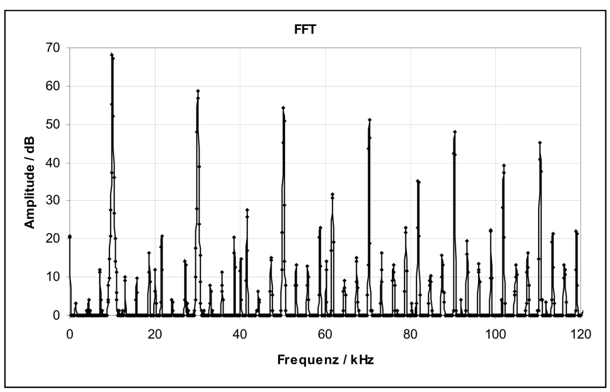
Eidgenössische Technische Hochschule Zürich
Swiss Federal Institute of Technology Zurich

Manual No.:

Harmonic Analysis

Advanced Physics Laboratory
ETH Zurich





Author: Reiner Mühle September 2001
Translation: Colin v. Negenborn December 2010

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1 Introduction

For the transfer of messages via electromagnetic signals it is generally necessary to fit those signals to the properties of the transmission channel through modifications [3]. A wireless information transmission with reasonable complexity, for instance, is only possible above a certain frequency limit. The signal that shall be transmitted therefore has to be transposed from its original frequency to a higher frequency range. This signal conversion is called *modulation*.

By transposing the signal and transmitting it at a different frequency range one can, to some extent, also achieve an improvement in the signal-to-noise ratio.

The modulation applied at the sender's side of the channel needs to be reversed at the receiver's side through demodulation such that the transmitted signal is available in its original frequency range.

For the process of modulation, a carrier wave is influenced by the modulating signal. In the case of a sinusoidal carrier wave, the technique of "modulation" can be used to vary the parameters *amplitude*, *frequency* and *phase angle*. Thus, a distinction is drawn between amplitude (AM), frequency (FM) and phase modulation (PM). The last two always occur together and are also called angle modulation.

In the experiment at hand, the process of amplitude modulation will be is analysed in greater detail [1].

2 Tasks

In this experiment, a frequency analysis (Fourier analysis, harmonic analysis) of various signals (square, triangle, etc.) shall be carried out using a band-pass filter. For this purpose, the amplitude of a carrier wave is influenced by a signal's wave via a modulator. The results shall be compared to those from a Fourier analysis (FFT: Fast Fourier Transform) conducted on a digital oscilloscope, to those from a Fourier analysis of the digitally saved signal data, and to the coefficients calculated from the respective Fourier series.

2.1 Devices

For the experimental set-up, the following devices are available:

1. PC with GPIB card, LabView with software (Harmonic analysis), Wavestare for TDS210
2. 2 Keithley multimeters 2700
3. Wavetek 5 MHz function generator, model FG-5000 for carrier signal
4. Wavetek 4 MHz function generator, model 182A for modulation signal
5. 101 Pulse generator with variable pulse width [1]
6. pulse generator (self-construction) [1]
7. modulator
8. band-pass filter
9. rectifier
10. ramp generator
11. analogue two-channel oscilloscope, Tectronix 2205 20 MHz
12. digital oscilloscope, Tectronix TDS210, 60 MHz [2]

[1] only one device each for all workstations

2.2 Preparation

In order to carry out the experiment easily and efficiently, it is absolutely necessary to thoroughly and critically **work through** this experiment's manual and to gain an overview of the following key aspects with the literature denoted in section 7:

2.2.1 Frequency Analysis

- Fourier series:
  - calculation of the Fourier coefficients of signals that will be analysed in detail in the experiment
- Fourier transformation:
  - system analysis

2.2.2 Modulation Techniques

- Amplitude modulation:
  - double-sideband modulation with carrier
  - double-sideband modulation without carrier
  - single-sideband modulation
  - degree of modulation
  - balanced modulation
- Angle modulation:
  - frequency modulation
  - phase modulation
- Pulse modulation

2.2.3 Band-pass Filter

- High-pass
- Low-pass
- Band-pass
- Resonance frequency

- Q factor of the filter
- Bandwith

2.3 Acquired Knowledge

After completion of the experiment, one should have understood the following key aspects:

- Modulation techniques for telecommunication
- Fourier transform, Fourier series
- Fast-Fourier-Transform (FFT)
- Measured value acquisition (frequencies, alternating current voltage, direct current voltage)
- analogue oscilloscope
- digital oscilloscope

3 Theory

3.1 Fourier Spectra

Signals can be described both as a function of time and as function of the frequency. The latter is more abstract, but of equal value. The advantages of a description in the frequency domain are *inter alia*:

- elegant system analysis (frequency response, transfer function)
- examination of the spectral cleanness of the signal
- general signal analysis and signal synthesis
- conclusions about necessary bandwith for transmissions.

The transition from the time domain to the frequency domain is achieved with the Fourier transform:

$$F(\omega) = \int_{-\infty}^{+\infty} f(t) \cdot e^{-i\omega t} dt. \tag{1}$$

The inverse transformation gives us the signal as an integral of the amplitude density, multiplied by harmonic functions:

$$f(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(\omega) \cdot e^{i\omega t} d\omega. \tag{2}$$

For periodic signals, $f(t)$, the Fourier spectrum, $F(\omega)$, is discrete, i.e. only certain discrete frequencies exist. Thus, periodic signals of period T can be expanded as a series, the so-called Fourier series:

$$f(t) = \sum_{n=-\infty}^{+\infty} c_n \cdot e^{in\omega_0 t}, \tag{3}$$

with the coefficients given as

$$c_n = \frac{1}{T} \int_0^T f(t) \cdot e^{-in\omega_0 t} dt, \tag{4}$$

where ω_0 is the fundamental frequency in the time domain. In contrast to equation (1) one notices that only integer multiples of a fundamental frequency appear. If we consider the limit $T = 2\pi/\omega_0 \to \infty$, we can see that the discrete spectral lines move closer and closer until they form a continuous spectrum as in equation (1).

If $f(t)$ is an even function, $F(\omega)$ is real. If $f(t)$ is odd, however, $F(\omega)$ is purely imaginary. The same applies to the coefficients, c_n.

3.2 Transfer Functions of Linear Systems

3.2.1 General Comments

We consider a system where an input function, $f_{in}(t)$, generates an output function, $f_{out}(t)$, at the exit. The system shall be linear, i.e. the superposition principle can be applied. The output function of two arbitrary input functions is therefore equal to the sum of the output functions of the respective input functions. Thus, if an input signal has been dissected into its harmonic components by the use of a Fourier transform (or Fourier series), one can transfer every single component through the system and then superpose all output signals in the transmission.

The output signal, $f_{out}(t)$, can then be written as weighted sum of all input signals at different times, τ:

$$f_{out}(t) = \int_{-\infty}^{+\infty} g(t, \tau) \cdot f_{in}(t)\, d\tau \tag{5}$$

The function $g(t)$ is the weighting function of the system, i.e. describing the contribution of the input function, $f_{in}(t)$, to the output function, $f_{out}(t)$, at time, τ.

Applying the Fourier transform and transferring the system to the frequency domain, the output function's Fourier transform is now given by the simple algebraic equation

$$F_{out}(\omega) = G(\omega) \cdot F_{in}(\omega) \tag{6}$$

Figure 1 visualises the connection between the system description in the time and frequency domains.

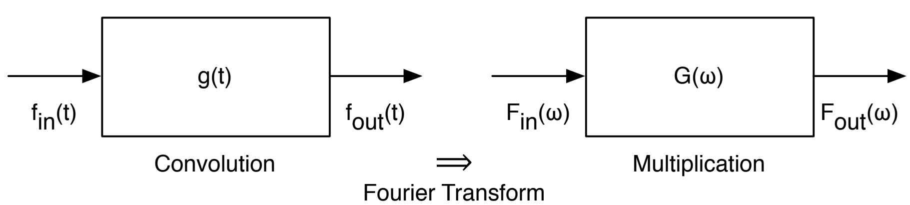

Figure 1: System description in the time and frequency domains

The function $G(\omega)$, being the Fourier transform of $g(t)$, describes the impact of the system on the harmonic signals of frequency, ω, and is also called the transfer function of the system.

$$G(\omega) = \frac{F_{out}(\omega)}{F_{in}(\omega)} = \mid G(\omega) \mid \cdot e^{i\phi(\omega)} \tag{7}$$

As is the case for any complex number, $G(\omega)$ is well-defined by its absolute value and phase and can be visualised as a vector in the complex plane. $|G(\omega)|$ then describes the vector's length and ϕ is the angle between the vector and the real axis. $|G(\omega)|$ is also called the system's amplification and is measured in decibel (dB):

$$V = 10 \cdot \log \left(\frac{F_{out}(\omega)}{F_{in}(\omega)} \right)^2 = 20 \cdot \log \left(|G(\omega)| \right), \tag{8}$$

$|G(\omega)|$ and ϕ can be derived from $G(\omega)$:

$$|G(\omega)| = \sqrt{[\Re(G)]^2 + [\Im(G)]^2} = \sqrt{GG^*} \tag{9}$$

$$\tan\phi = \frac{\Im(G)}{\Re(G)} \tag{10}$$

where we used

G^*	the complex conjugate of G,
$\Re(G)$	the real part of G,
$\Im(G)$	the imaginary part of G.

We will now compute the transfer functions for several systems which will be needed in the experiment at hand.

3.2.2 Low-pass

The low-pass filter is a circuit component which transfers low frequencies unaltered but damps the amplitudes of higher frequencies and shifts their phases. The set-up of a simple low-pass filter with resistor, R, and capacitor, C, is depicted in figure 2.

We derive the equation for the transfer function:

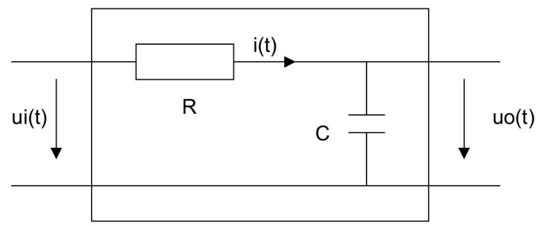

Figure 2: Passive low-pass

$$\begin{aligned} u_{out}(t) &= \frac{q(t)}{C} + u_{out}(0) = \frac{1}{C}\int i(t)\cdot dt + u_{out}(0) \\ &= \frac{1}{C}\int \frac{u_{in}(t)-u_{out}(t)}{R}\cdot dt + u_{out}(0) \\ \frac{d}{dt}u_{out}(t) &= \frac{1}{C}\left(\frac{u_{in}(t)-u_{out}(t)}{R}\right) \end{aligned}$$

The Fourier transform of both sides yields:

$$i\omega \cdot U_{out}(\omega) = \frac{1}{C}\left(\frac{U_{in}(t)-U_{out}(t)}{R}\right)$$

The transfer function of the filter is therefore given as:

$$G(\omega) = \frac{U_{out}(\omega)}{U_{in}(\omega)} = \frac{1}{1+i\omega RC} = |G(\omega)| \cdot e^{i\phi(\omega)} \tag{11}$$

From equation (11) one can see that higher frequencies are damped. In order to specify this proposition, we describe $G(\omega)$ in the so-called Bode plot. It consists of the two following illustrations:

1. Absolute value: $V = 20 \cdot \log|G(\omega)| = V[\log(\omega)]$
2. Phase: $\phi = \phi[\log(\omega)]$

In our example, we have

$$\begin{aligned} V &= -10 \cdot \log\left[1+(\omega RC)^2\right], \\ \phi &= -\arctan(\omega RC) \end{aligned} \tag{12}$$

The frequency where the amplification drops by the factor $\sqrt{2}$ or $3dB$, respectively, is called the cutoff frequency, ω_c. For the passive low-pass we get:

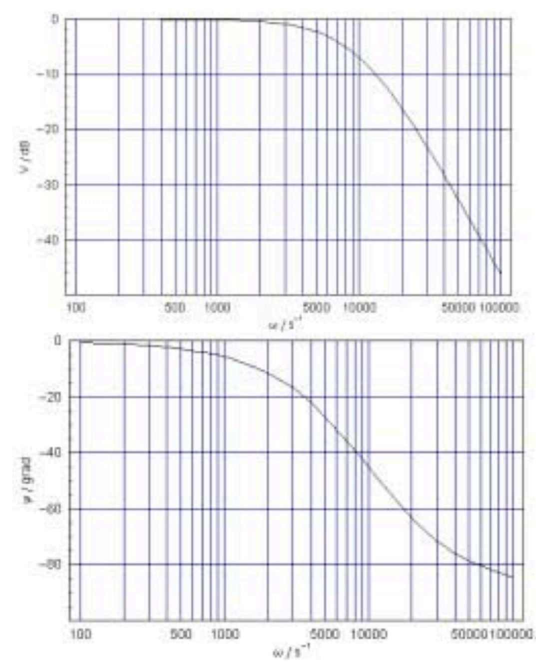

Figure 3: Bode plot for a passive low-pass ($R = 100\Omega$, $C = 1\mu F$)

$$\omega_c = \frac{1}{RC}$$

as can be derived from equations (11) and (15). By introducing a normalized complex frequency, P:

$$P = i\frac{\omega}{\omega_c} \tag{13}$$

we can write $G(\omega)$ in a more general form, which results in a clearer description for complicated systems:

$$G(P) = \frac{1}{1+P}$$

If n low-passes are connected in series, the frequency response has the following form:

$$G(P) = \frac{1}{(1+a\cdot P)(1+b\cdot P)(1+c\cdot P)\dots}$$

where the constants $a, b, c, \dots$ denote the respective cutoff frequencies of the particular low-passes. This equation can be rewritten in the following general form:

$$G(P) = \frac{1}{1+\sum_{i=1}^{n} a_i P^i}$$

Here, the a_i are the positive, real coefficients and n (the highest power of P), is the order of the filter.

3.2.3 High-pass

A basic passive high-pass filter is depicted in figure 4. It transfers high frequencies unaltered but damps the amplitudes of lower frequencies and shifts their phases, as seen in figure 5.

The transfer function, $G(\omega)$, of the high-pass filter can then be written as:

$$G(\omega) = \frac{i\omega RV}{1+i\omega RC} = \frac{P}{1+P} = \frac{1}{1+1/P} \tag{14}$$

The amplification and phase are therefore given by:

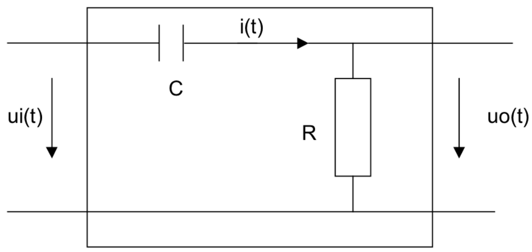

Figure 4: Passive high-pass

$$
\begin{aligned}
V &= -10 \cdot \log \left[1 + \left(\frac{1}{\omega RC} \right)^2 \right], \\
\phi &= -\arctan\left(\frac{1}{\omega RC} \right)
\end{aligned}
\tag{15}
$$

If n high-passes are connected in series, the frequency response (14) has the following form:

$$
G(P) = \frac{1}{1 + \sum_{i=1}^{n} a_i P^{-i}} \tag{16}
$$

3.2.4 Band-pass

By series connection of a high-pass filter and a low-pass filter , one can create a band-pass filter. In order to produce the simple multiplication of the transfer functions, the series connection has to be carried out non-reactively. This can achieved, e.g., by including operational amplifiers (op-amp) as impedance converters, see figure 6. This way, an active amplification could also be implemented.

The transfer functions $G_L(\omega)$ for the low-pass and $G_H(\omega)$ for the high-pass are given as:

$$
G_L(\omega) = \frac{1}{1 + i\omega R_L C_L} \tag{17}
$$

$$
G_H(\omega) = \frac{i\omega R_H C_H}{1 + i\omega R_H C_H} \tag{18}
$$

Multiplication yields the G_{BP} for the band-pass:

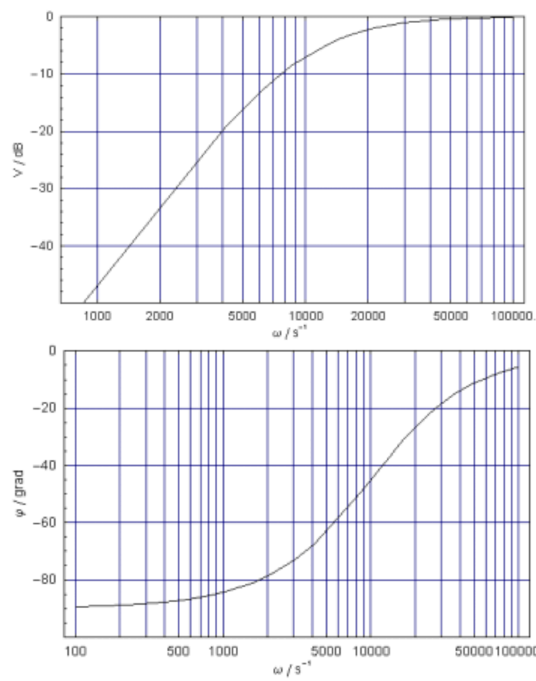

Figure 5: Bode plot for a passive high-pass ($R = 100\Omega$, $C = 1\mu F$)

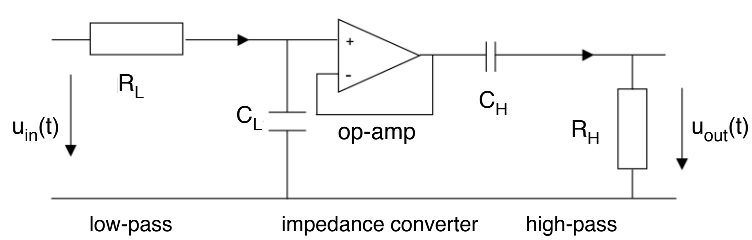

Figure 6: Passive band-pass

$$G_{BP}\left(\omega\right)=\frac{i\omega R_HC_H}{1+i\omega\left(R_HC_H+R_LC_L\right)-\omega^2R_HC_HR_LC_L}. \tag{19}$$

With

ω_0		angular resonance frequency,
P	$=i\frac{\omega}{\omega_0}$	normalized angular frequency,
τ_H	$=R_HC_H$	high-pass time constant,
τ_L	$=R_LC_L$	low-pass time constant,

equation 19 changes to

$$G_{BP}(P)=\frac{\omega_0\tau_HP}{1+\omega_0\left(\tau_H+\tau_L\right)P+\omega_0^2\tau_H\tau_LP^2}\equiv\frac{aP}{1+bP+cP^2} \tag{20}$$

At the resonance frequency ω_0, $\left|G\left(\omega\right)\right|$ has a maximum, $P=i$. The following equation has to hold:

$$\frac{\partial\left|G(\omega)\right|}{\partial\omega}\quad\text{at}\quad\omega=\omega_0$$

This is only possible for $c=1$. Therefore, $G(\omega_0)$ is real and the amplification at resonance is given by:

$$G_0=G(\omega_0)=\frac{a}{b} \tag{21}$$

3.3 Amplitude Modulation

For the amplitude modulation – as the name implies –, the amplitude of a high-frequency carrier signal, $u_C(t)$, is being modified in the rhythm of a low-frequency modulation signal, $u_M(t)$. The easiest device to achieve this is a analogue multiplier (see annex for the internal circuit) as depicted in figure 7.

If both input signals are harmonic, the output signal of an ideal modulator is also harmonic. KA is an offset which raises or lowers the carrier signal and m is the degree of modulation, characterizing the strength of the modulation. It is being varied by adding an offset KB to the modulation signal. Both quantities can be adjusted through potentiometers at the modulator. The modulator has an amplification, α, with a constant value of approximately 0.1 for the device at hand.

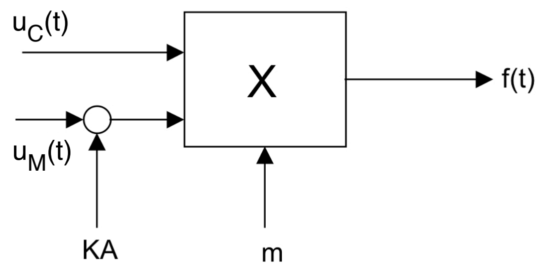

Figure 7: Analogue multiplier as modulator

With the input signals

$$u_C(t) = A + \hat{u}_C \cos(\omega_C \cdot t) \tag{22}$$

for the carrier signal and

$$u_M(t) = B + \hat{u}_M \cos(\omega_M \cdot t) \tag{23}$$

for the modulation signal, the output signal after the modulator is given as

$$\begin{aligned} f(t) &= \alpha \cdot [A + KA + \hat{u}_C \cdot \cos(\omega_C \cdot t)] \cdot [B + KB + \hat{u}_M \cdot \cos(\omega_M \cdot t)] \\ f(t) &\equiv \alpha \cdot \left[A' + \hat{u}_C \cdot \cos(\omega_C \cdot t)\right] \cdot \left[B' + \hat{u}_M \cdot \cos(\omega_M \cdot t)\right] \end{aligned} \tag{24}$$

where we used the the abbreviations $A' = A + KA$ and $B' = B + KB$.

The signals are characterized by the following quantities:

$\hat{u}_T$	and $\hat{u}_M$	signal amplitudes
$\omega_T = 2\pi f_T$	and $\omega_M = 2\pi f_M$	angular frequencies
$T_T = \frac{1}{f_T}$	and $T_M = \frac{1}{f_M}$	periodic time of the signals.

Thus, one receives an amplitude-modulated oscillation. Figures 8 and 9 show the amplitude modulation of two harmonic signals for visualization. The effects of a change in the constants KA and KB are illustrated.

For both signals the following quantities were assumed:

$$\omega_M = \frac{2\pi}{T_M},\ \omega_C = 20\omega_M,\ \hat{u}_M = 0.6,\ \hat{u}_T = 0.8.$$

In figure 8, the modulation was set for the values $A' = 0.5$ and $B' = 1$. In figure 9, the constant KA was changed such that $A' = 0$ was set. This

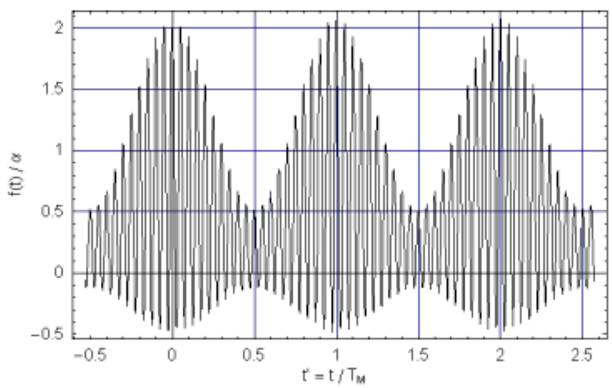

Figure 8: Amplitude modulation for $A' = 0.5$, $B' = 1$

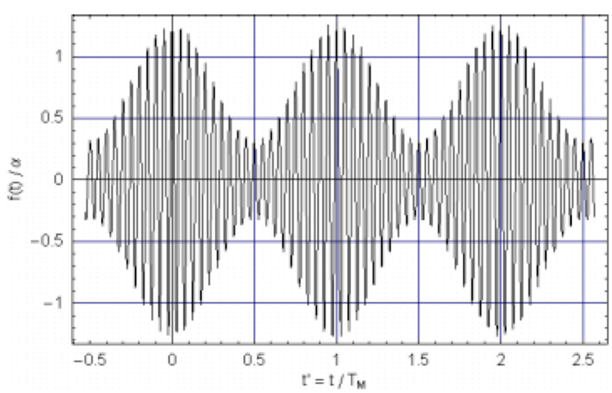

Figure 9: Amplitude modulation for $A' = 0.5$, $B' = 1$

yields the following expression for the modulated oscillation's output signal (with $A' = 0$):

$$\begin{aligned} f(t) &\equiv \alpha \cdot \hat{u}_C \cdot \big(B' + \hat{u}_M \cos\omega_C t\big) \cdot \cos\omega_C t \\ &= \alpha \cdot \hat{u}_C \cdot \big(B' \cos\omega_C t + \hat{u}_M \cos\omega_M t \cdot \cos\omega_C t\big) \end{aligned} \tag{25}$$

This expression can be rewritten as

$$f\left(t\right) = \alpha \cdot \hat{u}_C \cdot \left\{ B' \cos\omega_C t + \frac{\hat{u}_M}{2} \left[\cos\left(\omega_C + \omega_M\right) t + \cos\left(\omega_C - \omega_M\right) t\right] \right\} \tag{26}$$

One notices that besides the carrier frequency, ω_C, two other frequencies appear, symmetrically arranged around the carrier frequency (see figure 10).

In most cases, not only one specific signal frequency is being transmitted, but a whole frequency band. Therefore, one speaks about sidebands occurring in the spectrum symmetrically around the carrier frequency, called the upper sideband (USB) and the lower sideband (LSB) respectively. By transposing the signal's frequency band to the carrier's frequency range, the upper sideband appears in the so-called normal position and the lower sideband in the inverted position.

Both side frequencies contain all information of the modulation signal, apart from the phase. The envelope $u_E(t)$ of the amplitude-modulated oscillation (i.e. the progression of the carrier oscillation's amplitude, dependent on the modulation signal) follows the rhythm of the signal oscillation.

Denoting the carrier amplitude's oscillation around the original value, $\hat{u}'_C = \alpha \cdot \hat{u}_C \cdot B'$, as amplitude swing, $\Delta\hat{u}_C$, one can (for $B' \neq 0$) define the degree of modulation m as

$$m = \frac{\Delta\hat{u}_C}{\hat{u}'_C} = \frac{\hat{u}_M / B'}{\alpha \cdot \hat{u}_C \cdot B'} = \frac{\hat{u}_M}{\hat{u}_C} \frac{1}{\alpha \cdot B'^2} \tag{27}$$

Therewith, the envelope's equation is given as

$$\begin{aligned} u_E(t) &= \alpha \cdot \hat{u}_C \cdot \big[B' + \hat{u}_M \cos\omega_m t\big] \\ &= \alpha \cdot \hat{u}_C \cdot B' \left[1 + \frac{\hat{u}_M}{B'} \cos\omega_M t\right] = \hat{u}'_C \left[1 + m \cdot \cos\omega_M t\right] . \end{aligned} \tag{28}$$

The experimental value of degree of modulation can be determined either via the time function or the spectrum of the amplitude-modulated oscillation. For this purpose, the modulating signal, $\hat{u}_M$, is fed to the oscilloscope

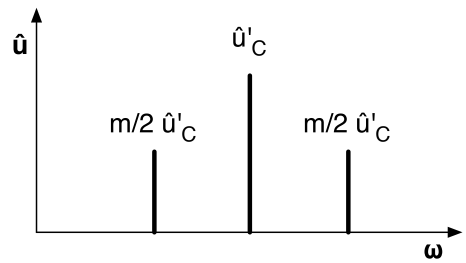

Figure 10: Spectrum of the amplitude-modulated oscillation

as the horizontal deflection (x-axis) and the modulated signal, $f(t)$, as the vertical deflection (y-axis). The triggering is carried out with $\hat{u}_M$. In the standard time display, m can be calculated via the enveloping curve's minimum and maximum value:

$$\begin{aligned} A_{max} &= \hat{u}'_{C_{max}} &= \hat{u}'_C \cdot (1+m)\,, \\ A_{min} &= \hat{u}'_{C_{min}} &= \hat{u}'_C \cdot (1-m)\ . \end{aligned}$$

Therefore, the degree of modulation can be derived from the oscillation's plot as

$$m = \frac{A_{max} - A_{min}}{A_{max} + A_{min}} \tag{29}$$

Particularly for the control of a constantly varying degree of modulation, m can be determined from the so-called modulation trapezium. One uses the x-y-display at the oscilloscope. The resulting oscillogram shows a trapezium with the sides $2 \cdot A_{max}$ and $2 \cdot A_{min}$ given that there is no phase shift between the envelope and the modulation signal, as displayed in figure 11.

Thus, for $\alpha = 1$ and $B' = 1$ the degree of modulation is given as the ratio of the signal amplitude to the unmodified carrier amplitude, as seen in equation 27.

In figure 12 the modulated signal is displayed for a degree of modulation $m = 1$. The curves with side frequencies, $\omega_C \pm \omega_M$, thus have half the intensity of the central curves with carrier frequency, ω_C, in this case.

From the frequency spectrum one can also calculate the power, P_{AM}, of the amplitude-modulated oscillation, related to the power, P_C, of the unmodulated carrier oscillation:

$$P_{AM} = P_C + P_{USB} + P_{LSB} = P_C \cdot \left(1 + \frac{m^2}{4} + \frac{m^2}{4}\right)\ . \tag{30}$$

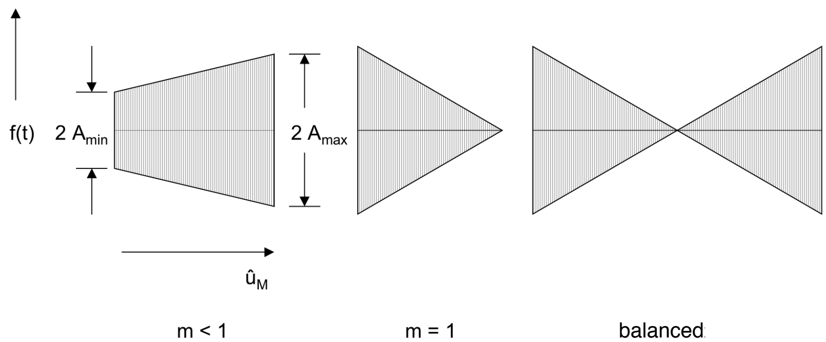

Figure 11: Modulation trapezium for different values of the degree of modulation m

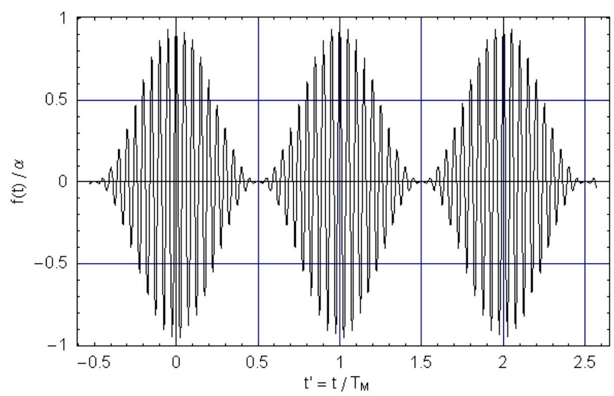

Figure 12: Amplitude modulation for $m = 1$

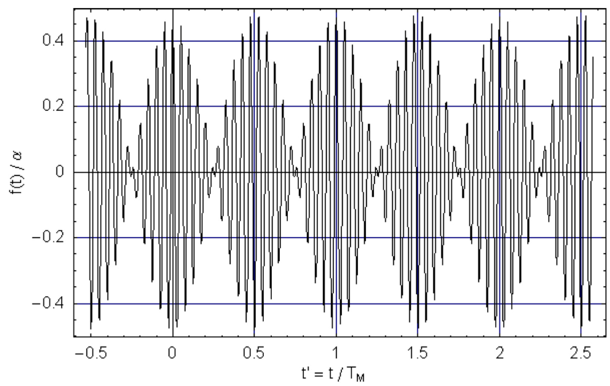

Figure 13: Amplitude modulation for balance modulation (without carrier signal

This result can easily be derived from the calculation of the power in the alternating current circuit. It follows that for full modulation ($m = 1$), half of the total power has to be invested into the carrier frequency. In this case, the total power of the sender increases to up to 1.5 times the carrier power. Therefore, one generally tries to totally suppress the carrier frequency at the sender's side and then add it at the receiver's side. This process is called balanced modulation. As visible in equation (26), this can be achieved via $B' = B + KB = 0$. We therefore change KB (potentiometer "degree of modulation") until the carrier frequency vanishes.

The output signal for balanced modulation is then given as

$$f(t) \equiv \alpha \cdot \hat{u}_C \cdot \hat{u}_M \cos \omega_M t \cdot \cos \omega_C t \,. \tag{31}$$

The result is displayed in figure 13. The x-y-plotting at the oscilloscope yields a curve symmetrical about the y-axis, see also figure 11.

3.4 Total Harmonic Distortion

The total harmonic distortion (THD) indicates the quality of an harmonic signal. It is defined as

$$K = \sqrt{\frac{\sum_{n=2} c_n^2}{c_1^2}} \tag{32}$$

The c_n denote the amplitudes in the Fourier spectrum (c_1: fundamental oscillation). Thus, if a signal only consists of a single harmonic (fundamental) oscillation, its THD is equal to zero. The bigger the THD, the worse the signal's quality. In acoustic engineering, a sound with $K \leq 1\%$ is described as good.

3.5 Discrete Fourier Transform

With the digital oscilloscope TDS 210, the plotted signal can be saved digitally. The chosen time scale, B, is divided into $n = 2,500$ intervals and saved as a table with the pairs of variates, (t_r, a_r), where $r = 1, \ldots, n$. The quantity, a_r, is the signal's value at time, t_r.

There are various definitions of a Fourier transform. In Mathematica, the Fourier transform, b_s, is defined as a list, a_r, of length, n, with the following expression:

$$b_s = \frac{1}{\sqrt{n}} \sum_{r=1}^{n} a_r e^{2\pi i \frac{(r-1)(s-1)}{n}} . \tag{33}$$

One has to consider that the value of b_s can be complex (convert to $|\, b_s \,|$) and that the term belonging to zero frequency appears at position 1 in the resulting list.

As the screen is 10 units wide, the total time scale covers $B_g = 10 \cdot B$. The corresponding frequency interval per point is then given as $\frac{1}{B_g}$.

For $B = 250\mu s$ we have

$$\frac{1}{B_g} = \frac{1}{2500\mu s} = \frac{4}{10^3 \cdot 10^{-6} s} = 4 \frac{kHz}{\text{point}} .$$

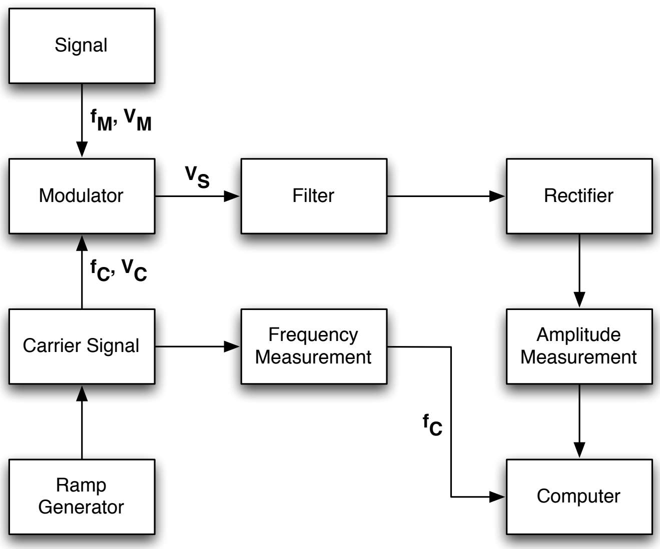

Figure 14: Sketch of the experimental set-up for amplitude modulation

4 The Experimental Set-Up

4.1 General Circuit Diagram

In order to measure specific frequency components of a spectrum, electronic analysers are especially appropriate. The best frequency resolution can be achieved with an heterodyne analyser. Such devices operate similarly to a radio receiver. The general circuit diagram is displayed in figure 14. The signal to be analysed (with frequency, f_M) is fed to a modulator, where it modulates the amplitude of a carrier signal (with higher, variable frequency f_C). At the modulator's output, the spectrum of the lower-frequency signal, appears as the lower and upper side band of the carrier signal. Thus, the information of the modulation signal is available in the modulated signal twice.

After the modulator, a filter of fixed resonance frequency, f_R, is connected. By varying the carrier frequency, this allows a sampling scan of the concurrently changing spectrum. After rectifying the signal with a precision rectifier, the amplitude can be measured and plotted on the oscilloscope or the computer.

4.2 Description of the Specific Components

The circuits of the specific electronic components are listed in the annex.

4.2.1 Modulator

In the experiment, an analog multiplier as described in 3.3 is being used. The amplification factor, α, is 0.1. The offset voltages, KA and KB, for carrier and modulation signal can be changed by means of potentiometers.

4.2.2 Filter

A band-pass filter of third order is used. The Q factor is given as approximately 100, the resonance frequency as $90kHz$. The filter's Q factor can be increased internally but decreases the range of linear amplification.

4.2.3 Ramp Generator

The ramp generator creates a voltage which linearly increases with time. This voltage controls the frequency, f_C, of the Voltage Controlled Oscillator (VCO). It is also used for synchronisation of the x-y-plotting at the oscilloscope (see Figure 14). The output voltage, U_{out}, can be changed through the variable potentiometer "voltage" in a range of $0-10V$. The time constant can be set through the potentiometer "time" in a range of approximately $30-330s$. The mutual dependence between the potentiometer adjustment and the time is not linear.

If the ramp generator is started, the frequency of the test signal generator with chosen time constant is linearly increased from the starting frequency, f_S, to the final frequency. f_F:

$$f_F = f_S + B \cdot V_A \tag{34}$$

B is a constant with a value of value of about 0.11 kHz/mV. With the switch "MAN", the measurement range can be tested roughly.

4.2.4 Rectifier

In the set-up at hand, an electronic rectifier is being used. Its amplification is constant at a large range of the incoming voltage, V_{in}, i.e. the output voltage, V_{out}, of the precision rectifier as a function of V_{in} can be written as

$$V_{out} = K_1 \cdot |V_{in}| \quad \text{and} \quad V_{out} = K_{10} \cdot |V_{in}|, \tag{35}$$

where $K_{10} = 10 \cdot K_1$.

This means that for small signals the amplification can be increased to achieve better measurements without distorting the data

5 The Experimental Procedure

In order to achieve optimal results, it is important to test all electronic devices of the set-up individually after a certain heating period. This way, potentially necessary adjustments can be applied to the final results and an overmodulation of the components can be prevented.

5.1 Characteristics of the Rectifier

- Determine the rectifier's transmission characteristics and derive the constants, K_1 and K_{10}, from the measurements.
- Calculate the possible range for the input voltage.

5.2 Characteristics of the Filter

- Check the range of V_{in} where the filter has a constant amplification. Plot both the input and output signal at the KO (at a frequency close to resonance, f_R) and examine the change in both signals for a change in V_{in}.
- Plot the band filter's resonance curve and determine the bandwith, B, and the Q factor, Q,. Take care that the filter is not driven into saturation.

5.3 Balanced Modulation

- Measure the frequency spectrum of an amplitude-modulated signal. Choose a sinusoidal signal and watch the effect of a change in the modulation frequency, f_M, the degree of modulation, m, and the offset, OS.
- Determine the degree of modulation in three ways: via an amplitude analysis on the KO, via measuring the trapezium in the x-y-plot (see figure 11) and via amplitude comparison in the frequency spectrum (see figure 10).

- Adjust the modulator to balanced modulation for the further measurements.

5.4 Finding Fourier Spectra

Take the Fourier spectra of some periodic signals and compare the results from the different measurements:

- band filter
- FFT analysis with the digital oscilloscope (amplitudes are given in dB!)
- Fourier analysis of the digitally saved signal (e.g. in Mathematica)

The analysis of rectangular signals, positive pulses with varying duty cycles and rectified sinusoidal signals is especially interesting.

Compare the measured Fourier amplitudes with the calculated values from the series expansion and discuss the cause of possibly occurring differences.

5.5 Determining the Total Harmonic Distortion

Determine the THD of a sinusoidal signal from a signal generator of low quality.

6 Annex (Circuit Diagrams of the Devices)

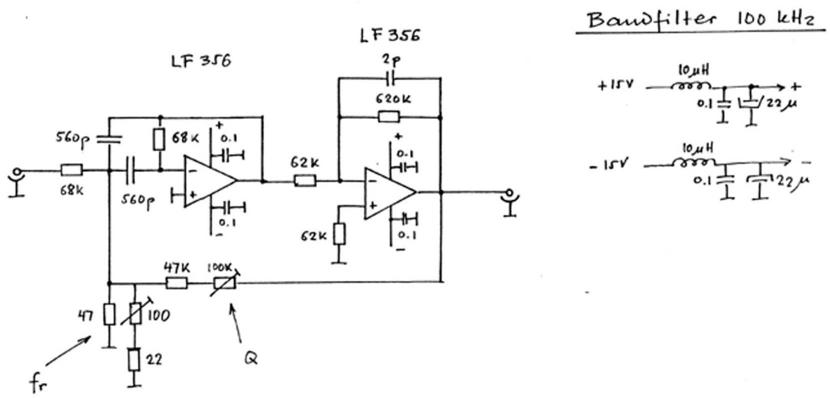

Figure 15: Band filter

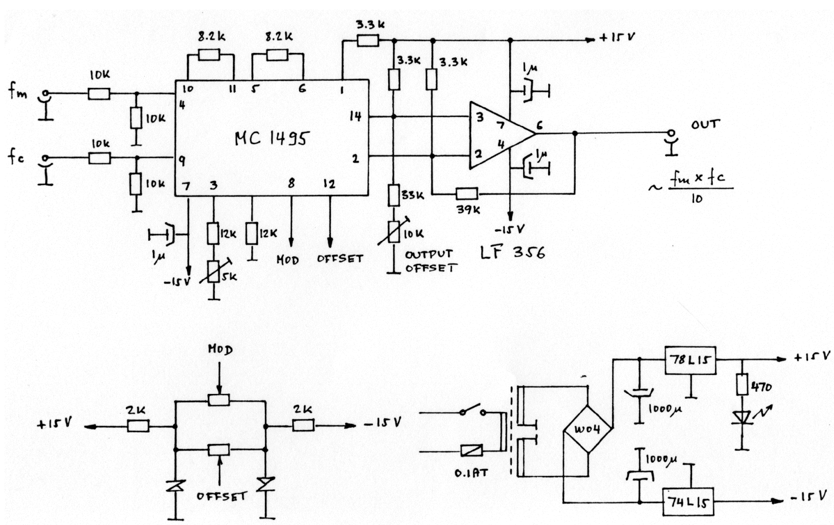

Figure 16: Modulator

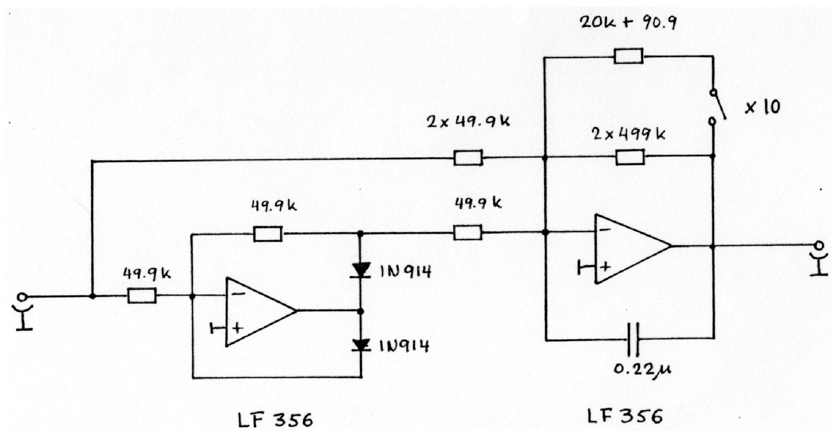

Figure 17: Rectifier

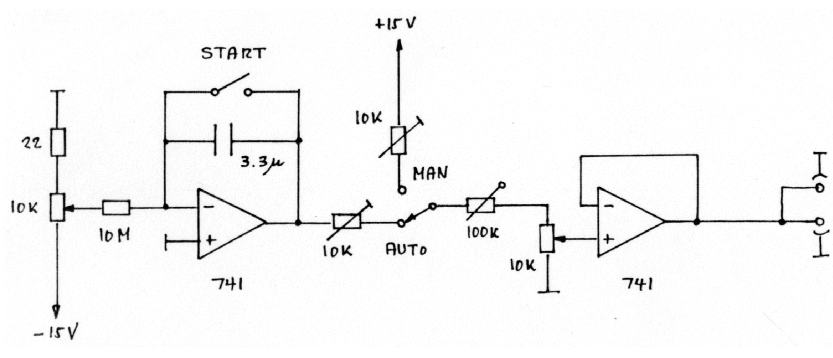

Figure 18: Ramp generator

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